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The effect of temperature on the coefficient of elasticity of a spring: construction of a device for its determination and calculation of its internal energy as a training practice

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Abstract

An experimental work carried out within the framework of research on the development of devices for teacher training in physics at the National University of General Sarmiento is presented. The device allows the determination of the coefficient of elasticity '*K*' of a metallic spring when its temperature is modified. The variation of the coefficient has as a direct consequence a variation in the internal energy. It is established, from the theoretical model, the way to calculate it. Finally, the construction of the device used is described and the results obtained are presented.

Keywords: coefficient of elasticity, internal energy, construction of devices, teacher training in physics

(Some figures may appear in colour only in the online journal)

1. Introduction

Among the traditional laboratory practices in subjects in the area of physics, particularly in Newtonian mechanics, are those that consider the stretching of a spring against the action of a force. If it is operated within the range where elasticity is guaranteed, a stretch is obtained that has an essentially linear relationship with the applied force. Typically, the assembly is carried out on a support with a spring suspended from one of its ends and different masses are placed on its free end, measuring the elongation they produce. In this way, the aforementioned linearity is established by a scalar that is usually called the elastic constant or coefficient of elasticity. This constant depends on several intrinsic characteristics of the spring such as its length, and number of turns, among others [1]. The relationship between stretch and applied force is known as Hooke's law [2].

That linearity, fully accepted from the classic teaching situations, is satisfactory within certain limitations of the model itself. In addition to the most obvious, which consists in not stretching the spring beyond the point from which it shows permanent deformation, there is another one, very rarely considered, which is the influence of temperature on the test. Specifically, it is the elasticity coefficient that would be significantly affected by this variable.

In our case, at the National University of General Sarmiento, we have considered this aspect in a space of teacher training in physics. It is an advanced subject for university professors in physics where experimental practices are designed in the laboratory, but with the purpose of carrying them out with non-commercial devices and even with common elements. In addition, this work is part of a research project that addresses the possibilities of developing didactic material for the laboratory with accessible materials. When these are also everyday objects, they can be very useful for disciplinary didactics [3]. In this way, the experimental development described here falls under the objectives of said research.

2. Theoretical considerations

We will initially consider a spring that obeys Hooke's law of elasticity, where y is the elastic force, K is the coefficient of elasticity of the spring, and x is its stretch

$$y = K.x. \tag{1}$$

This is a typical problem in classical mechanics, which is usually solved without reference to temperature. However, for real materials, the elasticity constant K depends on the temperature. Taking this fact into account, and neglecting the thermal expansion of the spring, it is possible to calculate its change in entropy with a change in its temperature.

2.1. Determination of entropy, S

Since the stretching of the spring can be considered a reversible process, the physical entropy in its classical form is defined by the Clausius function [4], which depends only on the initial and final states of the system

$$\mathrm{d}s = \frac{\mathrm{d}Q}{T}. \tag{2}$$

And since $\mathrm{d}Q$ represents an infinitesimal variation of heat in the spring, we can say that

$$\mathrm{d}Q = C_x \mathrm{d}T = T\mathrm{d}s, \tag{3}$$

where C_x is the heat capacity of the spring. Thus, the derivative of entropy with respect to temperature is

$$\left.\frac{\partial S}{\partial T}\right|_x = \frac{C_x}{T}. \tag{4}$$

Now, if we consider that the total derivative of the entropy is

$$\mathrm{d}S = \left.\frac{\partial S}{\partial T}\right|_x \mathrm{d}T + \left.\frac{\partial S}{\partial x}\right|_T \mathrm{d}x. \tag{5}$$

And, taking into account the relation of equation (4) we can write

$$\mathrm{d}S = \frac{C_x}{T}\mathrm{d}T + \left.\frac{\partial S}{\partial x}\right|_T \mathrm{d}x. \tag{6}$$

Here, we must recall the set of thermodynamic equations derived from Clairaut's theorem and the definitions of the thermodynamic potentials. From these equations, known as Maxwell's relations [3], we can use that

$$\left.\frac{\partial S}{\partial V}\right|_T = -\left.\frac{\partial P}{\partial T}\right|_V. \tag{7}$$

And, establishing by analogy $V \to x$, and that $P \to y$, with which equation (7) results:

$$\left.\frac{\partial S}{\partial x}\right|_T = -\left.\frac{\partial y}{\partial T}\right|_x. \tag{8}$$

Considering now the relationship established in equation (1) by Hooke's law, and substituting in the equation (8)

$$\left.\frac{\partial S}{\partial x}\right|_T = -\left.\frac{\partial y}{\partial T}\right|_x = -\left.\frac{\partial (Kx)}{\partial T}\right|_x. \tag{9}$$

With which an infinitesimal change in entropy of the spring can be written as

$$\mathrm{d}S = \frac{C_x}{T}\mathrm{d}T - \frac{\partial (Kx)}{\partial T}\mathrm{d}x. \tag{10}$$

Integrating equation (10) and making the approximation that the heat capacity of the spring, C_x, does not depend on temperature and that the change in length of the spring due to its expansion is negligible (for the range of temperature we use, 50 K, the expansion was 0.4%), we can write

$$\int \mathrm{d}S = C_x \int_{T_0}^{T} \frac{\mathrm{d}T}{T} - \frac{\mathrm{d}K}{\mathrm{d}T}\int_{x_0}^{x} x\mathrm{d}x. \tag{11}$$

Finally, the entropy of the spring as a function of its temperature and its stretching is

$$S_{(T,x)} = C_x \ln\left(\frac{T}{T_0}\right) - \frac{\mathrm{d}K}{\mathrm{d}T}\frac{x^2}{2} + cte. \tag{12}$$

Where, naturally, the integration constant does not have a meaning of interest in the work because the variation of entropy is the relevant aspect.

2.2. Determination of the Helmholtz free energy, F

Now, taking into account the calculation of the entropy, the Helmholtz free energy can be obtained by calculating the potential energy of the spring and the amount of heat exchanged by the system, being defined as:

$$\mathrm{d}F = \mathrm{d}L - S\mathrm{d}T = y\mathrm{d}x - S\mathrm{d}t. \tag{13}$$

On the other hand, the total Helmholtz free energy differential can be written as

$$\mathrm{d}F = \left.\frac{\partial F}{\partial x}\right|_T \mathrm{d}x + \left.\frac{\partial F}{\partial T}\right|_x \mathrm{d}T. \tag{14}$$

Integrating equation (14) to both members, we have

$$\int \mathrm{d}F = \int \frac{\partial F}{\partial x}\bigg|_T \mathrm{d}x + \int \frac{\partial F}{\partial T}\bigg|_x \mathrm{d}T. \tag{15}$$

As mentioned above, in our case we can neglect the variation in the length of the spring due to its thermal expansion, with which $\int \frac{\partial F}{\partial x}\Big|_T \mathrm{d}x \approx 0$, then equation (15) can be written as

$$F = \int \frac{\partial F}{\partial T}\bigg|_x \mathrm{d}T = -\int S\mathrm{d}t. \tag{16}$$

Then, using the expression for entropy from equation (11), we can finally find the Helmholtz free energy

$$F = \int -C_x \ln\left(\frac{T}{T_0}\right)\mathrm{d}T + \int \frac{\mathrm{d}K}{\mathrm{d}T}\frac{x^2}{2}\mathrm{d}T \tag{17}$$

$$F = \frac{1}{2}Kx^2 - C_x T\left(ln\left(\frac{T}{T_0}\right) - 1\right). \tag{18}$$

2.3. Determination of internal energy, U

To determine the internal energy U, we start from the definition of the Helmholtz free energy

$$F = U - T.S \;\Rightarrow U_{(T,\,x)} = F_{(T,\,x)} + T.S_{(T,\,x)}. \tag{19}$$

With which, using equations (12) and (18), the internal energy is:

$$U_{(T,x)} = \frac{1}{2}Kx^2 - C_x T\left(ln\left(\frac{T}{T_0}\right) - 1\right) + C_x T\ \ln\left(\frac{T}{T_0}\right) - \frac{\mathrm{d}K}{\mathrm{d}T}\frac{Tx^2}{2} + \mathrm{cte}. \tag{20}$$

Finally, the internal energy U is determined by:

$$U_{(T,x)} = C_x T + \frac{1}{2}Kx^2 \; - \frac{1}{2}Tx^2\frac{\mathrm{d}K}{\mathrm{d}T}. \tag{21}$$

It can be seen in equation (21) that there is a first term corresponding to the energy acquired by the spring during heating, the second corresponding to the elastic potential energy and the third where the dependence on temperature, stretching and also the variation of the coefficient of elasticity K. This last term is the one that is not usually considered in training practices. The same, effectively, is of little value, however, in certain cases, it can become relevant.

Our experimental work was developed in order to determine, in the case of a specific spring, the variation of K present in the third term of that equation.

3. Experimental device

The constructed device consists of a vertical frame that allows the spring under study to be hung from one of its ends, thus leaving the other free to suspend different masses. The stretch is determined by visual comparison with a precision ruler arranged vertically and parallel to the spring, very close to it. To achieve a precise stretch value, a binocular loupe was attached to the device; with its magnification it was possible to determine by visual estimation between the ruler marks, displacements of a few tenths of a millimeter. This resource also required a

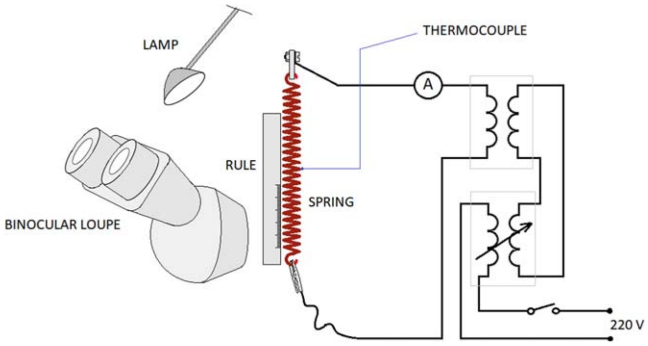

Figure 1. Scheme of the experimental setup.

concentrated lighting system in order to have more lighting, as in any optical instrument, that magnifies small details, in addition to seeking greater contrast according to its direction.

The controlled heating of the spring was precisely achieved by passing a variable electrical current at will. In our case, no additional resistor was added to the spring. In this way, the low resistivity of steel only allows voltages of relatively low values between the ends of the spring (of only a few Volts). And, as is known, it is the electric current that produces the heating and the temperature will depend on the section of the wire. The assembly to achieve these conditions consisted of a variable transformer 0–220 V (1 kW) to whose secondary a second transformer 220–30 V (30 A) was connected. With the manual variation that the first transformer allowed, a voltage between 0 and 30 V was achieved in the secondary of the second. Although the equipment could supply a maximum of 30 A, the working temperature values chosen for the spring used did not require currents greater than 5 A (at about 12 V). The spring was chosen among those available on the market, it has an outer diameter of 17.5 mm and 132 coils of 1.5 mm diameter wire.

It should be noted that tests were carried out to record the increase in length of the spring due to its own expansion (in a vertical position without suspended mass). This increase was practically undetectable with the method used, something predictable (and calculable) due to the relatively low temperatures used in the tests (less than 80 °C).

On the other hand, we have taken into account the attraction between turns due to the magnetic field that produces the circulation of the current. The result produced would be a shortening of the spring, that is, the opposite effect to that caused by the variation of the elastic constant due to an increase in temperature. However, for the values of the electric current of our tests it can be neglected. The calculation, in a simplified model of two closed and parallel turns, normal to the axial direction [5], a situation that overestimates the effect, yields a value less than 0.2 mm for the total length of the spring.

Figure 1 shows a schematic of the experimental setup.

The temperature in all the measurements was taken through a thermocouple in contact with of the spring in the middle of its length. The second method of measurement was photography with an infrared camera, which was also used to confirm that the temperature of the spring was homogeneous and stabilized in order to measure the change in the total length of the spring. It should be clarified that the general construction of the device was achieved to a certain extent with the reuse of elements, several of them being easily accessible materials.

The uncertainties in the measurements that we took for the calculations were 0.1 K (specific to the instrument) for the temperature and 0.2 mm for the stretching of the spring

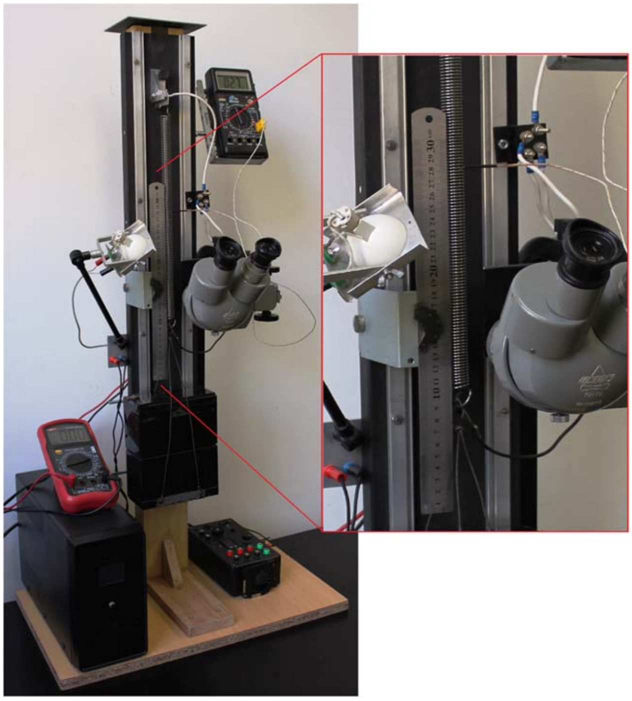

Figure 2. Photograph of the experimental setup of the device used for the measurements.

(visual magnification of its image next to the precision ruler). Figure 2 shows the experimental setup of the measurement device.

4. Results and discussion

As anticipated, advanced students of the physics teaching staff were made to participate in the tests in a teaching situation in the laboratory. Different considerations were taken in relation to the conditions of design and assembly of the device, taking into account the teaching competencies expected in the students of the teaching staff. Naturally, the theoretical aspects and the purpose of the experiment were previously discussed.

The determination of the elasticity coefficient K was carried out for different temperatures. Figure 3 shows two of the data fits obtained for temperatures of 303 K (room temperature) and 333 K; the linear dependence between the displacement and the applied force for both temperatures is observed in them, according to Hooke's law.

At the time of making the measurements, it was necessary to stabilize the temperature of the spring in its different values in order to measure its stretching. This procedure allowed to make the graphs of K versus the variation of temperature T. In order to determine if the temperature of the spring was stable and homogeneous at the time of carrying out the measurements, as mentioned above, a Fluke Ti450 infrared camera was used. Figure 4 shows two infrared thermographic images of the spring where the homogeneity in its temperature can be seen.

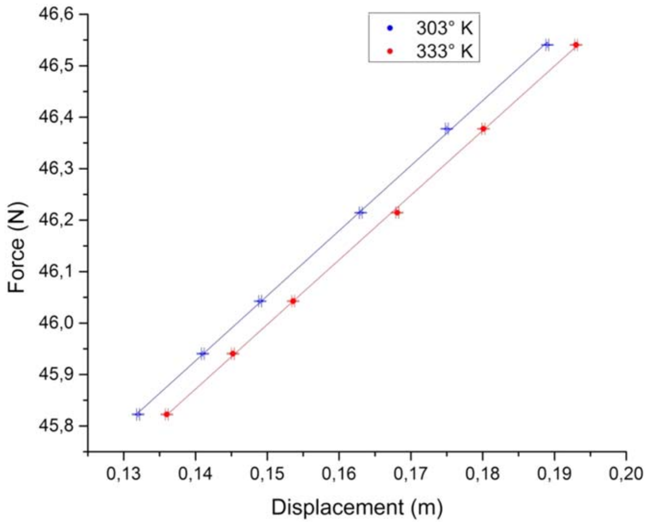

Figure 3. Linear adjustment of the force applied on the spring versus its stretching. It corresponds to two different temperatures, 303 K (blue) and 333 K (red).

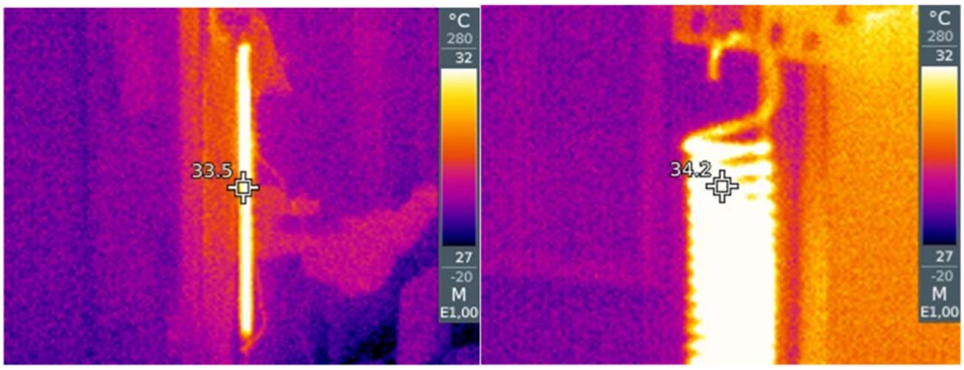

Figure 4. Infrared spring photographs. Left, entire spring, temperature homogeneity (33.5 °C) can be seen during the heating process. Right, details of the upper end of the spring where its coils and its fastening system can be seen in greater detail (temperature relieved: 34.2 °C).

A first observation that emerges from the work is that the method used turned out to be very precise despite the relative simplicity of the assembly. It was possible without difficulty to determine the stretches, that were clearly visible even though very small through the binocular loupe. Figure 5 shows an image captured through an eyepiece of the loupe.

Six series of measurements were made with different masses for the same temperature range. Figure 6 shows an example of the results obtained with the device built for the case of a force of 45.8 N. Those corresponding to the others are omitted because they have a similar behavior.

It can be seen from figure 6 that the recorded data can be fitted appropriately by linear regression. According to equation 21, one of the terms of the spring energy depends on the

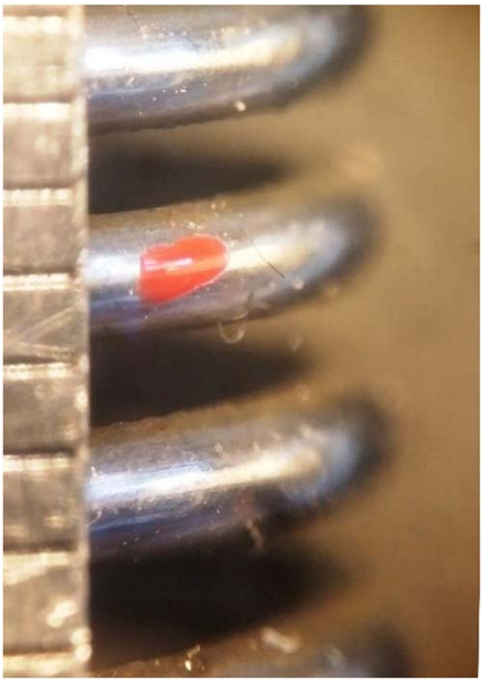

Figure 5. Image taken with camera through binocular loupe. The coils of the spring can be seen and to the left of the image the metal ruler used to measure the variations in length.

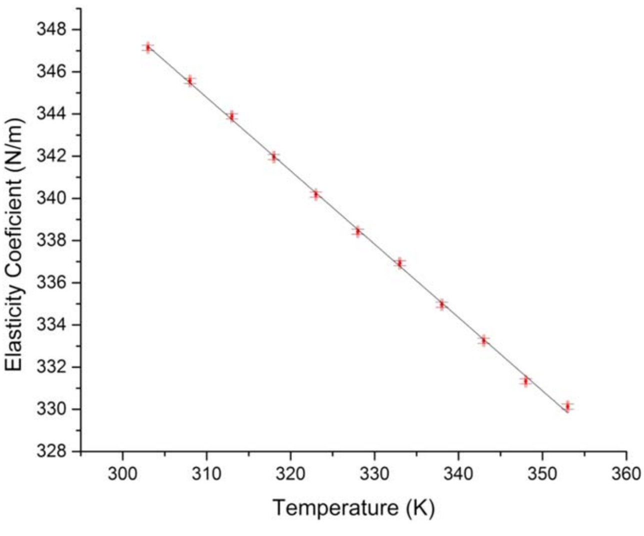

Figure 6. Linear adjustment of the variation of the coefficient of elasticity with temperature. It corresponds to a suspended mass of 4.67 kg.

derivative of the coefficient of elasticity K with respect to the temperature T, which corresponds to the slope of the function that approximates the points in figure 6.

Finally, we have considered the case of the two temperatures used in the graph of figure 3 for the calculation of the change in internal energy. The change under these conditions corresponds to 9% for a variation of only 30 K.

5. Conclusions

It is possible to determine the dependence of the elastic constant on temperature with a device built for the occasion, which can be assembled in a didactic laboratory, as was our case. We maintain that the practice was very suitable for working with students with whom not only the measurements were carried out, but also they had interventions in the fine-tuning of the device and its components. This allowed, on the one hand, to address a subject rarely taken into account in academic training, the variation of the elasticity constant K with temperature. On the other hand, it was possible to discuss with the students the feasibility of building devices whose design should be adjusted to the purposes of the experiment, taking into account the conditions that require a certain rigor in its construction. In addition, the device in question, with few modifications, could be useful for related tests that allow characterizing a spring in other aspects of interest for teaching. The calculation of energy, possible to carry out with a physical model that is finally expressed with a simple equation, provided an interesting framework for its conceptualization in a specific application context and, as already said, little considered in teacher training in physical sciences.

Finally, contributing to possible reproductions of the experience, it is convenient to mention that other previous test assemblies were discarded. One of them consisted of a horizontal arrangement of the spring with the masses suspended through a fine rope and a small pulley. It was inappropriate due to the presence of the latter, since it interfered with very small displacements (even with a very good quality bearing).

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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