Evolution of multifractal cross-correlations between Argentina MERVAL index and international commodities returns

Alejandra Figliola[†⋆] and Lucas Catalano[†‡]
[†] *Instituto del Desarrollo Humano, Universidad Nacional de General Sarmiento, J. M. Gutiérrez 1150, Provincia de Buenos Aires, Argentina.*
[⋆] *CONICET: Consejo Nacional de Investigaciones Científicas y Técnicas.*
[‡] *Dto. de Cs. Básicas, Universidad de Luján, Provincia de Buenos Aires, Argentina.*

We investigate the auto-correlations and cross-correlations of the volatility time series in the Argentina MERVAL index and commodity agricultural market, using the Detrended Cross-Correlation Analysis. We find that tax decisions implemented since 2004 and more so since 2009, produced a higher correlation between the indices of the MERVAL and the value of international grain prices.

Keywords: Detrended Cross Correlation Analysis ; Stock market commodities; Multifractal analysis; Time series analysis.

I. INTRODUCTION

Financial markets are highly complex systems that result from the interaction of many internal and external factors. For many years, economists, mathematicians and physicists have studied these time series, since their statistical properties are quite unique [1]. One of the first models of market prices was proposed by Louis Bachelier, who used a sequence of Gaussian random variables. In 1963, Mandelbrot contradicted Bachelier in a famous paper that showed that the series of returns for small intervals follows a Pareto-Levy distribution [2]. Years later, E. Fama introduced the *Efficient Market Hypothesis* (EMH), that states that at any time, the price of an asset fully reflects all avable historical information. This implies that there can be no correlation between past and future returns though it allows for correlations between their absolute values. More recently, physicists have used statistical physics and associated concepts such as “phase transitions” or “critical phenomena” to understand economic systems [3] and more specifically to propose some potentially universal properties of markets, so called *stylized facts*. One of the more remarkable stylized fact is that if one considers returns on different time scales, when the Δt increases the fat tail distribution, Pareto-Levy type, evolves towards a simpler Gaussian behavior. Another interesting stylized fact is the power law behavior of the autocorrelations of absolute returns,[2]. Also, recent works have demonstrated that empirical data coming from financial markets, such as stock market indices, foreign exchange markets, commodities, traded volumes and interest rates have a multifractal nature [4–9].

In this paper we intend to study the correlation between the series of international commodity prices and the evolution of Argentina’s economy in the period $2002 - 2012$. We will make use of the multifractal formalism to propose a specific way to estimate the cross-correlation between the MERVAL index and the commodity prices. This article is organized as follows: section II is a brief overview of the last economic history of Argentina; in section III we present the mathematical methodology, the next section we present and discuss the results and finally the conclusions.

II. BRIEF REVIEW OF RECENT ARGENTINIAN ECONOMIC HISTORY

Argentina is a country whose main activity is agriculture and livestock: it is one of the leading exporters of food commodities. From 1991 to 2001 -Menem’s and De la Rúa presidencies- the Argentine peso was at a fixed exchange rate with the US dollar. This “dollarization” was meant to overcome the tendency to periodic bursts of hyperinflation common during the late 1980s, but almost entirely deprived Argentina of any control over its monetary policy as it became evident when the aftermath of the “Tequila Crisis” provoked a massive outflow of capital. The 1997 financial crisis in SouthEast Asia caused a sudden revaluation of the dollar when compared with the currencies of competing countries and the impossibility to devalue the peso seriously harmed exports. As a result, the economy stopped growing, unemployment and poverty increased. social unrest grew and the country entered into a recession.

The crisis exploded on November 29, 2001, when Argentinians took to banks and financial institutions to withdraw millions of pesos and dollars from their accounts. Had the withdrawal continued, Argentina’s entire banking system would have collapsed. In this context, De la Rúa President’s position had become un-sustainable. Between December 16 and December 19 there were several incidents involving unemployed activists and protesters who demanded the

handing-out of food bags from supermarkets. These incidents ended up with outright looting of supermarkets and convenience stores on December 18, provoking 2 days later De la Rúa's resignation. Between December 20, 2001 and January 2, 2002 there were five presidents. The last, Eduardo Duhalde, called for elections won by Nestor Kirchner on May 2003. During the following year, Kirchner rescheduled 84 billion dollars in debts with international organizations, for three years. In the first half of 2005, the government launched a bond exchange to restructure approximately 81 billion of national public debt. Over 76 % of the debt was tendered and restructured for a recovery value of approximately one-third of its nominal value. These decisions as well as a large peso devaluation produced strong positive changes in the Argentinian economy and consequently in society. In October 2007, he was succeeded by his wife, Cristina Ferná ndez de Kirchner. who maintained the same course: economic development based on agro-industries, reduced debt and progressive social support to the most needy, in order to promote domestic consumption. Therefore, Argentina's economy is strongly linked to the export of raw materials, mainly soybeans, grains and beef. So, she urged the manufacturing industries for local development.
We consider these historical circumstances and divide the data set into three distinct periods T_1, T_2 and T_3, characterized by the political-economic contexts both internally and worldwide. From a domestic perspective, these periods represent essentially three presidential terms. Period T_1 : $2000 - 2004$, Fernando De La Rúa and Duhalde's government); T_2: $2004 - 2008$ Nestor Kirchner; T_3 $2008 - \textit{today}$ Cristina Fernández . Since 2007 the international price of some grains, especially soybeans, rose to record levels, while the period T_3 is characterized by a substantial increase in taxes on export of grains (especially soybeans), that the government used to reinvest into the economy through public works, subsidies and social programs.

III. MATHEMATICAL METHODS

A. The Multifractal Fluctuation Analysis

We will adopt the *Multifractal Detrended Fluctuation Analysis* (MFDFA) method, a generalization of the *Detrended Fluctuation Analysis* (DFA) method that has been proved to be a particularly flexible method, specially to deal with non-stationary series, [10, 11].

The MFDFA multifractal spectrum estimation of a one dimensional series $\{x(i), i = 1, \cdots, N\}$, is based on the construction and analysis of the *fluctuation function*, that is defined as a function of the *profile* of the series by the integration: $Y(k) = \sum_{i=1}^{k}[x(i) - < x >]$,: where $< x >$ is the mean value of the series $\{x(i)\}$. The *fluctuation function* is defined as:

$$F_s^2(\nu) = \frac{1}{s}\sum_{i=1}^{s} \{Y[(\nu-1)s+i] - p_\nu(i)\}^2 . \tag{1}$$

The profile is then cut into $N_s = N/s$ non overlapping segments of equal length s. The detrended time series for segment ν, denoted by $Y_\nu(i)$, is calculated as the difference between the original time series and a polynomial $p_\nu(i)$ that fits the series in the ν-th segment.

$$Y_\nu(i) = Y(i) - p_\nu(i) , \tag{2}$$

For simplicity we will use a polynomial fit of order 1, so that following the usual notation our algorithm is strictly the 1-MFDFA. In this paper we will use the simpler notation MFDFA, though we should keep in mind that different degrees in the polynomial mean different elimination of trends in the data. For each of the N_s segments, the variance of the detrended time series $Y_\nu(i)$ is evaluated by averaging over all data point i in the ν-th segment. Then, averaging over all segments, it is possible to obtain the q-th fluctuation function:

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s} \left[F_s^2(\nu)\right]^{q/2}\right\}^{1/q} , \tag{3}$$

where, in general, the index q can take any real value, and q work as a *mathematical microscope* that amplifies different behaviors of the data series, as we will show. The scaling behavior of the fluctuation function is determined by analyzing log-log plots $F_q(s)$ versus s for each value of q. If the series $x(i)$ is long-range power-law correlated $F_q(s)$ increases, for large values of s, as a power-law:

$$F_q(s) \sim s^{h(q)} . \tag{4}$$

For more details see [10].

For monofractal time series with compact support, $h(q)$ is independent of q, since the scaling behavior of the variance $F_s^2(\nu)$ is identical for all segments ν and the averaging procedure in Eq. 3 will give just this identical scaling behavior for all values of q. But if small and large fluctuations scale differently, there will be a significant dependence of $h(q)$ on q. If we consider positive values of q, the segments ν with large variance $F_s^2(\nu)$ will dominate the average $F_q(s)$ for sufficiently large q. Thus, for positive (large) values of q, $h(q)$ describes the scaling behavior of the segments with large fluctuations. On the contrary, for negative (large in absolute value) values of q, the segments ν with small variance $F_s^2(\nu)$ will dominate the average $F_q(s)$. Hence, for negative values of q, $h(q)$ describes the scaling behavior of the segments with small fluctuations and it is known as *generalized Hurst exponent.* When $q = 2$, the $h(2)$ is the *Hurst exponent.*

Following from Eqs. (3) and (4) and assuming that the length N of the series is an integer multiple of the scale s,

$$\sum_{\nu=1}^{N/s} \mid Y(\nu s) \; - \; Y((\nu-1)s) \mid^q \;\; \sim \;\; s^{qh(q)-1} \; . \tag{5}$$

Kantelhardt and co-workers argue that this multifractal formalism corresponds with the standard box counting theory and they related both formalisms. It is obvious that the term $\mid Y(\nu s) - Y((\nu-1)s) \mid$ is identical to the sum of the numbers $x(i)$ whit in each segment ν of size s. This sum is the box probability $p_s(\nu)$ in the standard formalism for normalized series $x(i)$.

The scaling function $\eta(q)$ is usually defined from last equation:

$$\eta(q) \; = \; q\, h(q) \; - \; 1 \tag{6}$$

where q is a real parameter. The Hölder exponent α and the multifractal spectrum $f(\alpha)$ are related with $\eta(q)$ via a Legendre transform, in the case that $\eta(q)$ is concave:

$$\alpha \; = \; \eta'(q) \tag{7}$$

and

$$f(\alpha) \; = \; q\, h \; - \; \eta(q) \; . \tag{8}$$

Then, MFDFA can be framed into the multifractal formalism. In multifractal systems, the strength of multifractality con be described by de width of the spectrum $\Delta\alpha$. It is easy to show that: $\alpha_{max} = h(-\infty)$ and $\alpha_{min} = h(+\infty)$. So, to estimate α_{max} and α_{min} we can use the function $h(q)$ with $\mid q \mid >> 1$.
For a stationary series as fractional Gaussian noise (fGn), the series (profile) is a fractional Brownian motion (fBm). For theses processes, $0 < h(q=2) < 1$ and $h(q=2)$ is the Hurst exponent, H. In the case of a monofractal signal with compact support, $h(q)$ is independent of q, because the scaling behavior of $F_q(s)$ is the same for all segments. Only if the little and big fluctuations in segments s whit a big variance the fluctuations scaled in a different way, the function $h(q)$ will depend with q, significantly.

B. The Detrended Cross-Correlations Analysis

Podobnik and Stanley [12] have proposed evaluate the *Cross-Fluctuation Function* based on in a work of Kantelhardt [10], who proposed the MFDFA estimator for the multifractal spectrum. Let two series with same length and sampled frequency ($\{s_1(i), i = 1, \cdots, N\}$ y $\{s_2(i), i = 1, \cdots, N\}$)

$$f^2_{MCCR}(\nu) = \frac{1}{r} \sum_{i=1}^{r} \; \{(Y_{1r}[(\nu-1)r+i])(Y_{2r}[(\nu-1)r+i])\} \tag{9}$$

when

$$Y_{1,2}(k) = \sum_{i=1}^{k} [s_{1,2}(i) - < s_{1,2} >]. \tag{10}$$

The estimator MF-CCR is the q-norm of $f^2_{MCCR}(\nu)$:

$$F_{MCCR}(q,r) = \left\{ \frac{1}{2N_r} \sum_{\nu=1}^{2N_r} \; \left[f_r^2(\nu)\right]^{q/2} \right\}^{1/q} \; . \tag{11}$$

When the series are non lineal cross correlated, they present a relation like:

$$F_{MCCR}(q,r) \propto r^{h_{MCCR}(q)}. \tag{12}$$

In the similar way that Eq. (4), the exponent $h_{MCCR}(q)$ can be obtained from the slope of the graph log-log of $F_{MCCR}(q,r)$ vs. r.
In the case $q = 2$ the cross correlation estimator $h_{MCCR}(q = 2) = h_{DCC}$ is known as Detrended Cross-Correlation. When $i = j$ the fluctuation function $F_{MCCR}(q,r)$ became the function $F_s(q)$ and $h_{MCCR}(q)$ became the standard *generalized Hurst exponent*, $h(q)$, and h_{DCC} became the Hurst exponent H.
The notions of persistence and anti-persistence are relevant to study the behavior of markets. These are used by market analysts to estimate future behavior when they want to make short-term estimates and these are related to Hurst exponent: $h(2)$. When the behavior is persistent, it holds that $0.5 < H < 1$ and when there is a higher probability that a positive trend (rise) will follow a rise and a negative (low), another low. In contrast, when the behavior is anti-persistent, the Hurst is between $0 < H < 0.5$ and they assume that an increase will most likely to be succeeded by a low and vice-versa. The $H = 0.5$ situation corresponds to an entirely uncorrelated behavior, so that it is not possible to estimate any trend.

The concepts of persistence and anti-persistence are directly generalized to the case of the cross-correlations between the fluctuation rates of a couples of series: the interpretation is similar to H: if $0 < h_{DCC} < 0.5$ correspond to the anti-persistent behavior, when one increases the other series decreases and in the case of persistent $0.5 < h_{DCC} < 1$, when one of the series grows (decreases) so does the other series. If $h_{CCD} = 0.5$ the two series are not correlated with each.
It is easy to show that, for the binomial multiplicative cascades with probabilities p_i y p_j and corresponded generalized Hurst exponent $h_i(q)$ and $h_j(q)$, the following equation is verified, [12]:

$$h_{MCCR}(q) = \frac{h_i(q) + h_j(q)}{2} \tag{13}$$

is interesting to note that in the case natural series, there is a relationship of inequality. For this, in this work, we calculate the coefficient proposed by [13]:

$$\mu_{i,j}(q) = (h_{MCCR}(q) - \frac{h_i(q) + hj(q)}{2})/h_{MCCR}(q) \tag{14}$$

For two binomial multiplicative cascades $\mu_{i,j}(q) = 0$ for all q, but, for other coupled of series $\mu_{i,j}(q)$ results a measure of the difference between the series and a process like a binomial cascades. The interpretation is: $\mu_{i,j}(q) > 0$ implies that the cross-correlations between the individual series are generally stronger than what would be expected from the long memory behavior of each of the individual series.

IV. DATA AND RESULTS

Were used the following data series: The *MERVAL Index* data that is an average of the top companies in the stock market in Buenos Aires and the international prices of three commodities: soybeans, corn and wheat. Data corresponding to the same period journals 1/8/2000 - 20/4/2012 and make a total of 2937 values for each series. Because particular countries schedules (national holidays, time off, etc.), it was necessary to add and remove any of the data in order to obtain series that dates coincide. It was derived from the data obtained for this commodity because these are international, in the event that the date of the Down Jones series is not in the number of commodities, this was removed, and if on the other hand, in the number of commodities is the data missing sometime in the Down Jones, this is added to the series by averaging between the previous day and the next. The three sub-series with the same length (979 data points) T_1, T_2 and T_3 corresponding the periods 1/8/2000 - 8/7/2004; 9/7/2004 - 3/6/2008 and 4/6/2008 - 20/4/2012 respectively. These series are synchronized with the corresponding Merval Index data.
Since the data series $\{x(i), i = 1, \cdots, N\}$ the returns coefficients are: $\{r(i) = log\frac{x(i)}{x(i+1)}, i = 1, \cdots, N-1\}$ and the volatility is the absolute values of the returns. In the Fig. 1, we present the normalized price series in this period for the Merval, soybeans, corn and wheat.

From these price series, we calculate the cross multifractal correlations between data sets MERVAL index returns and each of the series of returns of the index of prices of soybeans, wheat and corn. Using the formalism presented

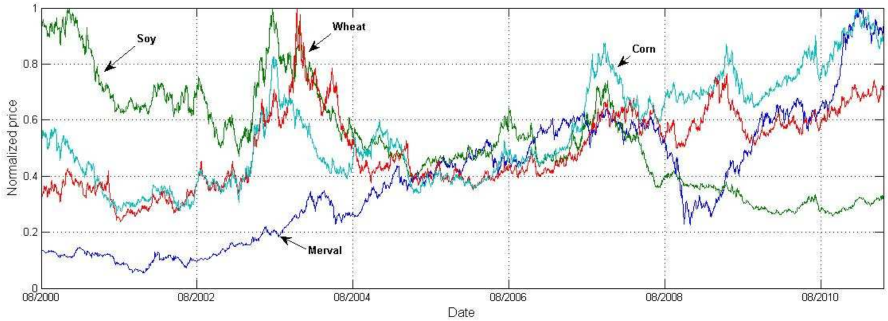

FIG. 1: Normalized prices for the Merval series and the international prices of soybean, wheat and corn.

above, we calculated the cross Hurst index $h_{MCCR}(q)$ from (12) and the coefficient $\mu_{i,j}$ from (14), for the three periods, T_1, T_2 and T_3. Table I present the results for the case $q = 2$ corresponding to the h_{DCC}.

Period	Couple of series	h_{ij}	$\frac{h_i+h_j}{2}$	% of μ_{ij}
T_1	MERVAL - Soy	0,53	0,53	0.8 %
T_1	MERVAL - Corn	0,55	0,52	3.9 %
T_1	MERVAL - Wheat	0,53	0,50	2.9 %
T_2	MERVAL - Soy	0,53	0,52	3.8 %
T_2	MERVAL - Corn	0,55	0,54	2.1 %
T_2	MERVAL - Wheat	0,50	0,50	0,4 %
T_3	MERVAL - Soy	0,63	0,59	6.8 %
T_3	MERVAL - Corn	0,59	0,59	4.8 %
T_3	MERVAL - Wheat	0,58	0,55	4.6 %

Tab. I: values of the cross correlation for the couples return prices of MERVAL Index and the international prices of soybean, corn ant wheat. Note that in all cases, $\mu \geq 0$, i.e. a persistence behavior.

Figures 2,3 and 4 show the features of fluctuation and cross fluctuation function for each of the time periods, T_1, T_2 and T_3, only for the case of the couple of MERVAL-Soybeans:

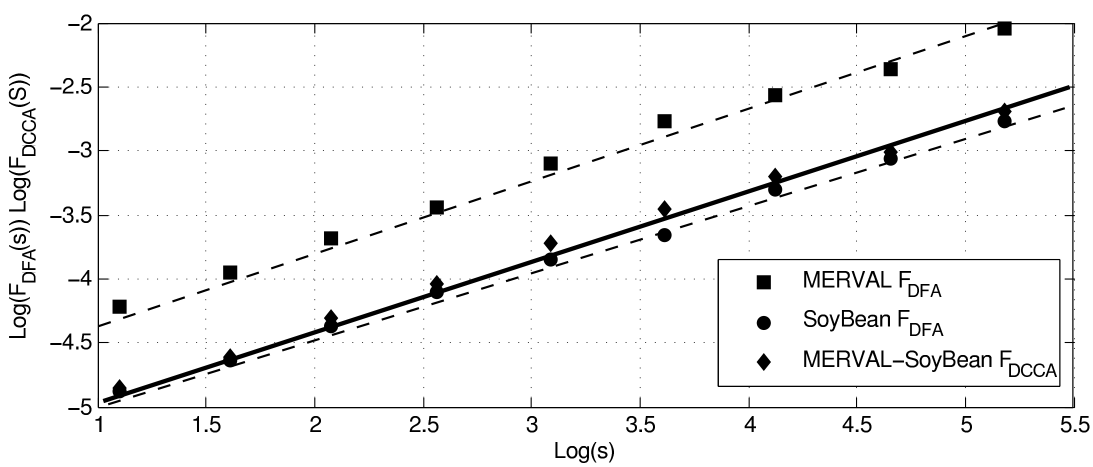

FIG. 2: Power law correlations and cross-correlations between MERVAL and soybean return time series, in the Period I.

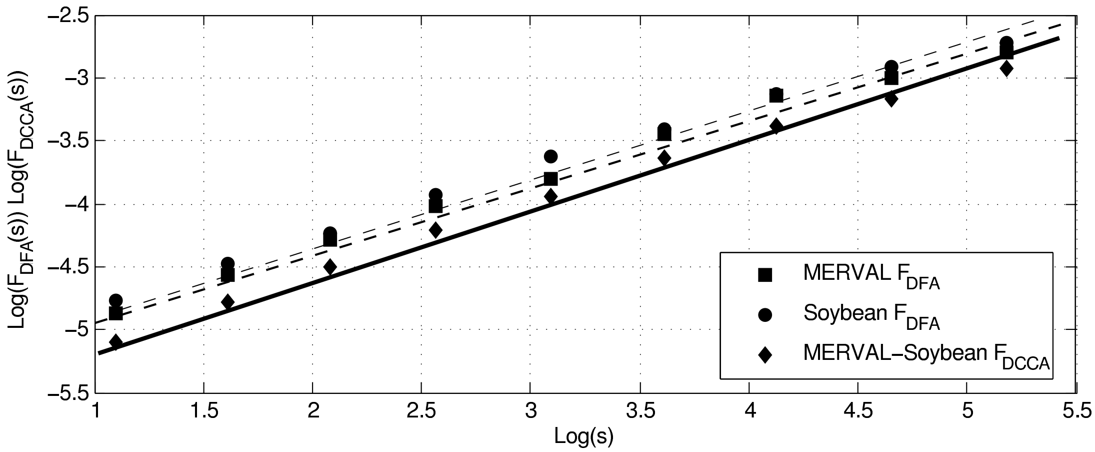

FIG. 3: Power law correlations and cross-correlations between MERVAL and soybean return time series, in the Period II. Cross-correlations are stronger than the individual auto-correlations.

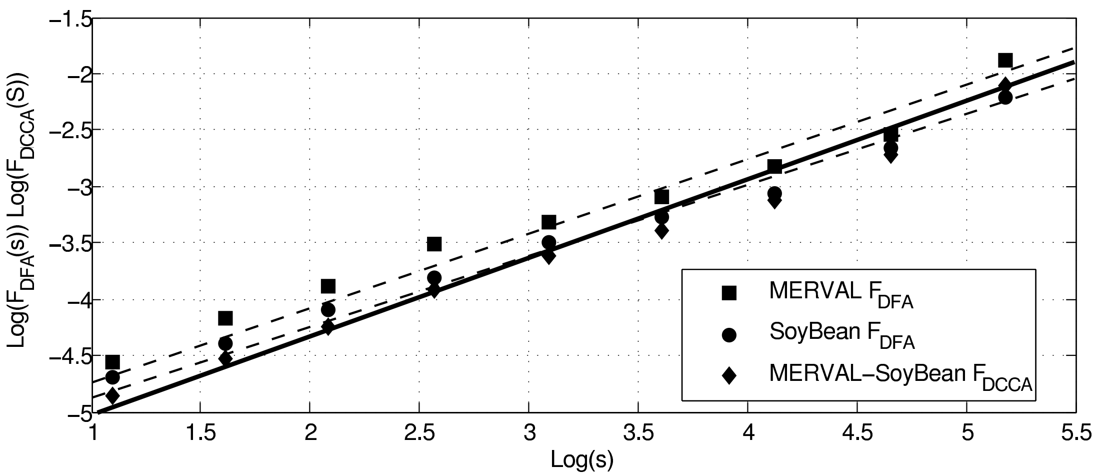

FIG. 4: Power law correlations and cross-correlations between MERVAL and soybean return time series, in the Period III. Cross-correlations are considerably stronger than the individual auto-correlations.

V. CONCLUSIONS

In this work we have investigated the relationship between the autocorrelations and cross-correlations in time series of returns for MERVAL and three series of agricultural commodities: soybeans, corn and wheat. We find that for the case of soybeans, in the first period cross-correlations were virtually identical to the autocorrelation ($\mu \approx 0$), but increased in the second period and reaches a maximum in the third period. In the case of corn and wheat, the second period has a lower cross-correlation (μ almost reaches zero for wheat) but becomes positive in the third period.

This result is fully consistent with the decisions taken from 2004 in respect to taxes on exports of grains and re-injection of the funds in social assistance (which increased the domestic market), construction loans and support industry.

[1] H. E. Stanley, V. Pleroua, X. Gabaix, “Statistical physics view of financial fluctuations: Evidence for scaling and universality”, Phys. A 387 (2008) 3967-3981.

[2] B. B. Mandelbrot, “The variation of certain speculative prices”, Journal of Business 36 (1963) 394–419.

[3] R. da Silva, M. Zembrzuski, F. Correa, L. C. Lamb, “Stock markets and criticality in the current economic crisis”, Phys. A 389 (2010) 5460–5467.

[4] B. B. Mandelbrot, “A Multifractal Walk down Wall Street”, Sci. Am. 280 (1999) 70–73.

[5] H.E. Stanley, L. A. Amaral, P. Gopikrishnan, P. Ch. Ivanov, T. H. Keitt and V. Plerou, “Scale invariance and universality: organizing principles in complex systems”, Phys. A 281(2000) 60–68.

[6] L. Zunino, B. Tabak, A. Figliola, D. G. Pérez, M. Garavaglia and O. A. Rosso, "A multifractal approach for the stock market inefficiency", Phys. A 387(2008) 26–37.
[7] L. Zunino, A. Figliola, B. M. Tabak, D. G. Pérez, M. Garavaglia and O. A. Rosso,"Multifractal structure in Latin-American market indices", Chaos, Solitons & Fractals. 6443, 41 (5) (2009) 2330–2339.
[8] A. Figliola, L. Zunino, E. Serrano and O. Rosso, "Multifractal characterization of the stock market inefficiency using the Wavelet Leaders method", Adv. & App. in Stat. Sci. 2, Issue 2 (2010) 287-302.
[9] K. Matia, Ashkenazy Y and Stanley H. E., "Multifractal properties of price fluctuations of stocks and commodities", Europhys. Lett. 61 (2003) 4228.
[10] J.W. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, H.E. Stanley, "Multifractal detrended fluctuation analysis of nonstationary time series", Physica A **316** (2002) 87–114.
[11] E. Serrano and A. Figliola, *Wavelet leaders: a new method to estimate the multifractal singularity spectra*, Physica (A) 388 (2009) pp. 2793–2805.
[12] B. Podobnik and E. Stanley "Detrended Cross-Correlation Analysis: A new method for analyzing two no stationary time series", Phys. Rev. Lett. **100**, (2008) 084102
[13] E. Leite Siqueira Jr., Tatijana Stošić, Lucian Bejan, Borko Stošić, "Correlations and cross-correlations in the Brazilian agrarian commodities and stocks", Physica A 389 (2010) 2739-2743